Terminal Velocity of Canola Oil, Hexane, and Gasoline Drops Rising in Water due to Buoyancy

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Abstract— Drops, globules of a liquid in another liquid, are extremely important in many natural processes and industrial applications. The purpose of this study was to devise a method to measure the terminal velocity of drops rising in water due to buoyancy, and to compare observed values with the theoretical. Two questions were explored: (1) Do these drops continue to accelerate upward from a depth of 6 cm; and, (2) Does the terminal velocity of these drops (modifying the experiment accordingly if not) match the theoretical (calculated) values? A syringe was used to inject 0.1 cm^3 (0.1 mL) drops of three liquids (oil, hexane, and gasoline) into a vessel at a depth of 6 cm, and the resulting motion was video captured and imported into the shareware kinematics program Tracker® for analysis and determination of terminal velocity. The experiments showed that the drops reached terminal velocity before reaching the surface (2.23 ± 0.10 cm, 1.48 ± 0.07 cm, and 1.35 ± 0.06 cm above the injection point, respectively). Secondly, in addition to the accepted term of πr^2 normally used for the projected area in the theoretical equation for terminal velocity, a new term, $2\pi r^2$, was also employed in order to account for drop flattening during ascension. As a result, the calculated value with the new term accurately predicted the observed, doing so better than the accepted term for all three liquids, and might be used to improve the accepted theory.

Index Terms—Acceleration, buoyancy, drag, drops, friction, projected area, spheres, terminal velocity.

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1 INTRODUCTION

Drops, globules of a liquid in another liquid, are of fundamental importance in many natural physical processes and in a host of industrial and man-related activities [1]. Rainfall, air pollution, boiling, flotation, fermentation, liquid-liquid extraction, and spray drying are only a few of the phenomena and operations in which drops as well as solid particles play a primary role. Meteorologists and geophysicists study the behavior of raindrops and hailstones. Applied mathematicians and physicists have long been concerned with fundamental aspects of fluid-particle interactions. Chemical and metallurgical engineers rely on drops for such operations as distillation, absorption, flotation, and spray drying. Mechanical engineers have studied droplet behavior in connection with combustion operations. In all these phenomena and processes, there is relative motion between bubbles, drops, or particles on the one hand, and surrounding fluid on the other [1].

While an understanding of drop behaviour is obviously valuable in real-world applications, there have been very few studies that explore the basics of drop motion, such as their response to the buoyant force. The theoretical principles have been documented [1], but actual experimental data are scant.

Therefore, the purpose of this work encompassed two questions relevant to drop behaviour: 1. Do drops of liquids less dense than water (canola oil, hexane, and gasoline) moving upward due to buoyancy continue to accelerate upward from a depth of 6 cm; and, 2. Will there be a difference between the observed and the theoretical values for the terminal velocity of the drops, and if so, can this be explained mathematically?

2 THEORETICAL BACKGROUND

2.1 Buoyancy

An object submerged in a fluid displaces a volume of fluid equal to the volume of the object itself, with the buoyant force $\vec{F}_b$ acting upon that object if it is less dense than the surrounding fluid [2]. The buoyant force is equal in magnitude to the weight of the displaced fluid, but opposite in direction; i.e., upwards. Thus, expressing mass as the product of density and volume, the buoyant force $\vec{F}_b$can be expressed by the formula:

$$\vec{F}_b = -\rho_f V \vec{g}\,, \tag{1}$$

in which $\rho_f$is the density of the surrounding fluid and V is the volume of both the displaced fluid and the immersed object, in this case a drop of liquid less dense than water ($\vec{g}$ = acceleration due to gravity). If the surrounding fluid is frictionless, the drop's acceleration $\vec{a}$ due to $\vec{F}_b$ is not proportional to the volume, as

$$\vec{a} = \frac{\vec{g}(\rho - \rho_f)}{\rho}\,, \tag{2}$$

where ρ is the density of the drop.

2.2 Drag and Terminal Velocity

However, only superfluids such as supercooled helium-2 are truly frictionless [3], [4]; therefore, modeling drop behaviour must consider drag $\vec{F}_d$, a second force acting on a drop.

$$\vec{F}_d = \frac{1}{2} C_d \rho_f v^2 A_p \tag{3}$$

In (3), $\vec{F}_d$ describes the force of the drag acting on an object

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(drop) in a liquid of some density ρ_f at some velocity v with a drag coefficient C_d, and projected area A_p [5]. For stable smooth spheres, the projected area A_p is the median cross-sectional area πr^2, and the drag coefficient C_d is 0.47 [5], [6]. Notably, an A_p of πr^2 assumes perfect spheres. However, we suspect that the drops might become flattened perpendicular to the direction of travel, becoming olblate spheroids. To accommodate for this, we propose that A_p would be better approximated by $2\pi r^2$, which would more accurately represent the cross-sectional area of a flattened drop. This treatment of A_p assumes the sphere becomes fully flattened, such that the cross-sectional area becomes ½ of the surface area of the sphere (i.e., ½ of $4\pi r^2$ thus $2\pi r^2$).

If drag $\vec{F}_d$ becomes large enough to exactly equal the drop weight plus the buoyant force, $\vec{F}_g + \vec{F}_b$, the net force will become zero, and acceleration $\vec{a}$ will cease (i.e., become zero as well). The drop will move at a constant velocity to the surface of the water, having reached terminal velocity, v_t:

$$v_t = \sqrt{\frac{2\vec{g}V(\rho-\rho_f)}{C_d \rho_f A_p}} \tag{4}$$

3 METHODS

3.1 Experimental

All experiments were performed at room temperature. A 1 cm^3 syringe with a 26-gauge needle was used to inject 0.1 cm^3 drops of canola (rapeseed) oil (density 0.92 g/cm^3 at room temperature [7]), hexane (density = 0.66 g/cm^3 at room temperature [7]), or standard (vehicle) gasoline "A" (0.74 g/cm^3 at room temperature [8]) into a straight-sided centimeter ruled plastic vessel filled with distilled water 6 cm below the surface. A digital video camera (Fuijifilm FinePix E900) operating at 30 frames per second was used to capture each drop as it rose in the water column. The experiment was repeated ten times for each liquid. The video data was imported into the freeware program Tracker® (Open Source Physics), and the kinematics of each liquid (velocity, acceleration) were determined. Simple statistics were used to evaluate the mean height above the injection point at which terminal velocity was reached for each liquid.

3.1 Calculations and Comparisons

Following the experimental determination of mean terminal velocity v_t for each liquid, the theoretical values were calculated from (4). As the volume V of a drop is known (0.1 cm^3), the radius r of the sphere to use in (4) for A_p (πr^2) was simply:

$$r = \sqrt[3]{\frac{3V}{4\pi}} \tag{5}$$

However, the drops did become flattened. To account for this phenomenon, a second value for A_p was also used in (4); namely, $2\pi r^2$.

4 RESULTS

4.1 Experimental

In all cases, drops became flattened perpendicular to the direction of travel, becoming olblate spheroids. Table 1 shows the mean values for terminal velocity v_t, height above injection site (position) when v_t was reached, and time when v_t was attained for the oil, hexane, and gasoline trials. Our novel experimental set-up revelaed that oil drops had the lowest terminal velocity, gasoline had second lowest, and hexane had the fastest. Oil drops reached terminal velocity last, while hexane reached terminal velocity first.

TABLE 1
TERMINAL VELOCITY, POSITION, AND TIME FOR OIL, HEXANE, AND GASOLINE

	Oil	Hexane	Gasoline
Mean terminal velocity, v_t (cm/s)	8.05 ± 0.05	13.15 ± 0.05	11.55 ± 0.05
Mean position when v_t is reached (cm above injection)	2.23 ± 0.10	1.48 ± 0.07	1.35 ± 0.06
Mean time when v_t is reached (s)	0.33 ± 0.01	0.13 ± 0.01	0.17 ± 0.01

Mean values for terminal velocity as well as position and time when terminal velocity was reached for rising oil, hexane, and gasoline drops (volume = 0.1 cm^3, n = 10).

4.2 Calculations and Comparisons

Table 2 compares the observed and predicted terminal velocities v_t for rising drops of oil, hexane, and gasoline. The observed values for v_t were much closer to the predicted values for all three liquids when $2\pi r^2$ was used for projected area A_p rather than the accepted πr^2.

TABLE 2
COMPARISON BETWEEN OBSERVED AND PREDICTED TERMINAL VELOCITIES

Liquid	Density (g/cm^3)	Terminal Velocity v_t (cm/s) Observed	Terminal Velocity v_t (cm/s) Predicted with $A_p = \pi r^2$	Terminal Velocity v_t (cm/s) Predicted with $A_p = 2\pi r^2$
Oil	0.92	8.05 ± 0.05	11.32	8.00
Hexane	0.66	13.15 ± 0.05	23.33	16.50
Gasoline	0.72	11.55 ± 0.05	20.41	14.43

Drop volumes were 0.1 cm^3, n = 10.

5 DISCUSSION

These results imply that there might be a better interpretation of the accepted formula used for predicting the terminal velocity of a sphere, if that sphere is a fluid that experiences compression during movement. Using $2\pi r^2$ for projected area A_p rather than the accepted πr^2 has the effect of taking ½ of the surface area of a sphere. Obviously, this assumes that the sphere is infinitely compressed, which of course is not the case. Nonetheless, $2\pi r^2$ is a better predictor of the experimentally-determined values of v_t. Alternatively, instead of manipulating the term for A_p, the drag coefficient C_d for a sphere, with a value of 0.47 [5], [6], could be adjusted to account for fluid sphere flattening, perhaps simply by multiplying C_d by a factor of 2: i.e., 2 x 0.47 = 0.94.

CONCLUSION

Considering the importance of drops in many natural processes and industrial applications, our revised term for determining projected area of a fluid compressible sphere, $A_p = 2\pi r^2$, or a modification of the drag coefficient C_d for flattening spheres should be of interest to many scientists and engineers who study drop behaviour.

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